Universitatea Babeş-Bolyai Cluj-Napoca
Facultatea de Matematică şi Informatică

Ciclul de studii: Doctorat
Domeniul: Matematica
Programul de studii: Școala Doctorală de Matematică
Limba de predare: Engleză

SYLLABUS

I. General data

Code	Subject
???	**Algebraic Groups**

Semester	Hours: C+S+L	Category	Status
2	2+2+0 (Alternatively, it may be offered as a reading course)	specialty	optional

II. Full status faculty members

Name and surname	Scientific title	Didactic title	Chair	Type of activity		
				C	S	L
MARCUS Andrei	Ph.D.	Prof.	Algebra	*	*	

Associated faculty members

Name and surname	Scientific title	Institution	Type of position	Type of activity		
				C	S	L

III. Course objectives

This course is an introduction to the study of algebraic groups, with a focus on those over fields of positive characteristic. Students will witness the interplay of methods from commutative algebra, topology, algebraic geometry, Lie algebras and finite groups. The course starts by introducing algebraic varieties and matrix groups, and the aim is to bring students to the point where they are able to read current research papers in the field, especially on the representation theory of algebraic groups and related finite groups. Homeworks include exercises of both theoretical and practical nature.

IV. Course contents

Algebraic sets. Regular maps. Algebraic groups. Lie algebra of a linear algebraic groups. Groups with a split BN-pair. Afine varieties. Birational equivalences. Group actions. Algebraic representations. Solvable groups. Parabolic subgroups. Borel subgroups. Frobenius maps. Finite groups of Lie type. Characters.

V. Bibliography

1. M. Geck, *An Introduction to Algebraic Geometry and Algebraic Groups*. Oxford University Press, 2003.

2. J.E. Humphreys *Linear algebraic groups*, 2nd ed. Springer-Verlag, Berlin, 1991.
3. T.A. Springer, *Linear algebraic groups*, 2nd ed. Birkhauser, Boston-Basel-Berlin, 1998.

VI. Thematic of didactic activities per weeks

Schedule of the courses and seminars

WEEK 1.
1. Algebraic sets and algebraic groups
   1.1. Affine spaces and the Zariski topology
   1.2. Ideals and Groebner Spaces.
   1.3. Hilbert polynomials and the dimension of an algebraic set

WEEK 2.
2. Tangents space and the Lie algebra of an algebraic group

WEEK 3.
3. Groups with a split *BN*-pair

WEEK 4.
4. Abstract affine varieties

WEEK 5.
5. Finite Morphisms and birational equivalences

WEEK 6.
6. Linearization of algebraic groups
   The unipotent variety of the special linear groups

WEEK 7.
7. Group actions on affine algebraic varieties

WEEK 8.
8. Algebraic representations, solvable groups and tori

WEEK 9.
9. Grassmanian varieties and flag varieties

WEEK 10.
10. Parabolic subgroups and Borel subgroups

WEEK 11.
11. Frobenius maps and rational structures

WEEK 12.
12. Frobenius maps and BN-pairs. Finite classical groups.

WEEK 13.
13. Deligne-Lustig varieties

WEEK 14.
14. The virtual characters of Deligne and Lusztig

VII. Didactic methods used

Lectures, presentations, conversations, projects, exercises, individual study, homework assignments.

VIII. Assessment

The course ends with a written exam. The exam subjects have theoretical questions and exercises. Additionally, students will have to write a survey paper (accompanied by a beamer presentation) on a more advanced topic. There is an evaluation of the overall seminar activity (including homeworks), as well.
The final grade is the mean of the grades mentioned above, according to the following rule:
The final grade = Homeworks 30%. Paper 40%. Written Exam 30%.

IX. Additional bibliography

Prerequisites

1. J.L. Alperin and R.B. Bell, *Groups and representations*. Springer-Verlag, New York, 1995.
2. K. Erdmann and M.J. Wildon, Introduction to Lie algebras, Springer-Verlag, London, 2006.
3. A. Marcus, *Algebra*. (in Hungarian) Cluj University Press, 2008. available at http://math.ubbcluj.ro/~marcus.
4. J.J. Rotman, *Advanced modern algebra*. 2nd ed. American Mathematical Society. Providence, RI, 2010.

Advanced

1. A. Borel, *Linear algebraic groups*, 2nd ed. Springer-Verlag, Berlin, 1991.
2. R.W. Carter, *Finite Groups of Lie Type: conjugacy classes and complex characters*, Wiley, New York, 1985.
3. R.W. Carter and M. Geck (eds), *Representations of Reductive Groups*, Cambridge University Press, Cambridge, 1998.
4. F. Digne and J. Michel, *Representations of finite groups of Lie type*, Cambridge University Press, Cambridge, 1991.
5. W. Fulton and J. Harris, *Representation theory, a first course*, Springer-Verlag, 1991.
6. M. Geck, D. Testerman and J. Thévenaz (eds.), *Group representation theory*. CRC Press Boca Raton, FL; EPFL Press, Lausanne, 2007.
7. L.C. Grove, *Classical groups and Geometric algebra*, American Mathematical Society, 2002.
8. J.E. Humphreys, *Introduction to Lie algebras and representation theory*, Springer-Verlag, Berlin, 1972.
9. J.E. Humphreys, *Reflexion groups and Coxeter groups*, Springer-Verlag, Berlin, 1990.
10. J.E. Humphreys, *Modular representations of finite groups of Lie type*. Cambridge University Press, Cambridge, 2006.
11. P. Tauvel and R.W.T. Yu, *Lie algebras and algebraic groups,* Springer-Verlag, Berlin, 2005.

Date,
2010, September 6

Course responsible,
Prof. MARCUS Andrei, Ph.D.

DEAN,
Prof. Leon Tâmbulea, PhD.

DOCTORAL SCHOOL COORDINATOR
Prof. Adrian Petrușel, Ph.D.